

The asymptotic error constant, i.e. the coefficient of  $\epsilon_n^4$  in the expansion of (1.2) can be obtained by retaining terms of  $O[\epsilon_n^4]$  in (2.1)–(2.4) and substituting in (1.2) as before. Using the relations (2.6) to simplify, the value of the asymptotic error constant is

$$\frac{1}{9}(21 - 8\theta) \frac{c_2^3}{c_1^3} - \frac{c_2 c_3}{c_1^2} + \frac{1}{9} \frac{c_4}{c_1},$$

where the tedious details of the calculation have been omitted. We see that the value  $\theta = \frac{2}{8}$  simplifies this constant by removing the first term. The corresponding values of the parameters are

$$a_1 = \frac{11}{28}, \quad a_2 = \frac{27}{52}, \quad b_1 = \frac{-1183}{64}, \quad b_2 = \frac{1911}{64}.$$

**3. Conclusions.** Four fourth order iterative formulae for solving equations have been derived which require one function and two derivative evaluations per iteration. It is interesting to note that these formulae form counterparts of the iteration function given by Traub [1, p. 184 (8–78)] which is of order 4 and uses two values of  $f$  and one of  $f'$  per iteration. The formulae obtained in this paper will be particularly appropriate for use in practical root finding problems where the derivative can be quickly computed compared with the function.

**4. Acknowledgment.** I am grateful to the referee for a number of valuable suggestions.

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1. J. F. TRAUB, *Iterative Methods for the Solution of Equations*, Prentice-Hall, Englewood Cliffs, N. J. MR 29 \*6607.

## Certification of Parlett's ALGOL Eigenvalue Procedure Eig 3\*

By J. M. Varah

The ALGOL program given by B. Parlett in [1] was tested on the Burroughs B5500 at Stanford University, in two ways. First, the program was checked for correctness of ALGOL 60 syntax, using a program due to William McKeeman [4]. Second, the program was modified to conform to Burroughs Extended ALGOL [3], roughly as in [2], and tested on several matrices. The following errors were found:

1. On p. 477, line 30, if  $B[1]$  is zero, a divide by zero may be encountered. If the eigenvalue iterate is real,  $B[4] = 0$  so that if  $B[1]$  is zero, control is transferred

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in line 29. However, if the iterate is complex,  $B [1]$  may be zero without control being transferred around line 30. But for a complex iterate, line 30 is superfluous, as other assignments are made in lines 31–35. So we can eliminate the divide by zero by changing lines 30 ff. to read:

```

if  $y = 0$  then
  begin
     $t1r := B[2]/B[1]; t1i := 0; t2r := B[3]/B[1]; t2i := 0$ 
  end
  else
    begin
       $t1r := (B[2] \times B[1] + B[5] \times B[4])/d1;$ 
      .....
    end
if; ... .

```

2. *Overflow* is defined as an external Boolean procedure but is assigned a value in procedure *Evaluate* (p. 477, line — 5). One way to correct this would be to define *overflow* as an external Boolean variable instead, which would be set to **true** if overflow occurred.

3. On p. 480, line — 4 should read:

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L4: for  $j := 1$  step 1 until  $m$  do  $spurc := spurc + RTR [j]; \dots$ 

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With these corrections, the program did run successfully on 50 matrices generated with eigenvalues at random from the Gaussian integers  $m + ni$ ,  $|m| \leq 50$ ,  $|n| \leq 50$ . At least 6 significant decimal digits were obtained in all cases on the Burroughs B5500, where the word length is about 11 decimal digits. The program also ran successfully on a number of ill-conditioned matrices.

Earlier tests of [1], reported in [2], failed to reveal error 1. The matter of overflow was understood in [2] and, of course, misprint 3 was not in the manuscript for [1].

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1. B. PARLETT, "Laguerre's method applied to the matrix eigenvalue problem," *Math. Comp.*, v. 18, 1964, pp. 464–485. MR 29 #2948.
2. G. E. FORSYTHE, "Tests of Parlett's ALGOL eigenvalue procedure Eig 3," *Math. Comp.*, v. 18, 1964, pp. 486–487. MR 29 #2949.
3. Burroughs Corporation, Equipment and Systems Marketing Division, "Burroughs B5500 Information Processing Systems Extended ALGOL Reference Manual," Detroit, Michigan, 1964.
4. W. M. McKEEMAN, Library Program #L4.080.A, Stanford Computation Center, Stanford, California.